TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 905

SOME INVESTIGATIONS OF THE GENERAL INSTABILITY OF STIFFENED METAL CYLINDERS

I - REVIEW OF THEORY AND BIBLIOGRAPHY

Guggenheim Aeronautical Laboratory
California Institute of Technology

Washington
July 1943

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INTRODUCTION

This is the first of a series of reports covering an investigation of the general instability problem by the California Institute of Technology. The first five reports of this series cover investigations of the general instability problem under the loading conditions of pure bending and were prepared under the sponsorship of the Civil Aeronautics Administration. The succeeding reports of this series cover the work done on other loading conditions under the sponsorship of the National Advisory Committee for Aeronautics.

The general instability project at the California Institute of Technology was initiated and sponsored by the Aircraft Engineering Division of the Civil Aeronautics Administration in 1938, for the purpose of obtaining design information for use in connection with extremely large airplanes. The first four reports of this series cover the results of the first year's research and were given limited distribution to manufacturers of large aircraft in 1939. The fifth report of this series covers the second year's research on the program which was carried out under the sponsorship of the Technical Development Division of the Civil Aeronautics Administration and was given limited distribution to the manufacturers of large aircraft in 1940.

In order to permit a better understanding of the work continued under the sponsorship of the NACA, it has been deemed desirable to publish the earlier reports along with the results of the current studies in the form of a series of reports covering the entire investigation.

STATEMENT OF PROBLEM

The rapidly increasing size of modern airplanes has brought forth a number of new structural design problems. One of these—is the problem of determining the allowable load that can be carried by stiffened cylinders of large radius. It is known that certain combinations of longitudinal stiffeners, bulkheads or frames, and sheet will give a cylinder that will fail in such a manner as to involve all three structural elements simultaneously. This type of failure has been called the general instability failure of the stiffened cylinders, and the determination of the parameters affecting the failing load of such a structure is a problem which is becoming more and more important to designers. The purpose of the general instability program was therefore to determine the following:

(a) A method of calculating the strength of cylindrical structures falling into the general instability classification

(b) The limits of the general instability regime.

(c) Design methods in any transition region which may occur between instability failures of a localized nature and those of a completely general type

In starting any research program such as this, it is first necessary to collect all of the available knowledge on the subject and to correlate the published theoretical and experimental results. A bibliography covering the strength properties of curved plates and shells was, therefore, compiled and the list of references that have been reviewed is attached to this report. This bibliography does not pretend to cover the entire field of thin-wall structures and contains only articles of some interest to the immediate research program. A number of the references are of only minor interest and are not considered in detail, however; some of them treat directly with our present problem and will be discussed at length later.

The review of the literature indicated that two things were vitally necessary to the success of the program. The first was the establishment of a nomenclature in which all terms were defined in such a manner as to be understandable by the average designer. The second, and by far the most important, was a systematic study of the fundamental principles upon which the whole conception of the failing strength of stiffened shells has been developed. The general instability failure of a stiffened cylinder will be shown to be only a small part of the general problem of the strength properties of stiffened shells, however; the basic ideas underlying the whole of the theory of shell structures are so interlocked that it is almost impossible to make a study of any isolated phase of the problem.

The body of this report will, therefore, consist of three parts. The first part will be devoted to the definition of terms and a general review of the problems involved in the study of the failures of thin stiffened shell structures. The second part will deal with the strength properties of unstiffened sheets and cylinders, primarily because those investigators who have worked on the problem of stiffened structures, have utilized the theories regarding unstiffened plates and shells as a background. A review and discussion of the available knowledge regarding the strength properties of stiffened cylinders which are subject to a general instability type of failure will make up the third section.

TYPES OF FAILURES IN STIFFENED SHELL STRUCTURES

One of the important elements of a typical metal airplane structure is the external skin which provides, first, an aerodynamical surface upon which the air forces act (in the case of the wings or control surfaces) and, second, a covering for the contents of the airplane (fuselage, fuel and oil tanks, etc.). In addition to these prosaic functions, the skin is also so designed that it is a load-resisting element, and as such, it acts as a part of the primary structure.

Some of the primary loads entering such a structure are obviously compressive in nature. Since thin sheet material is weak when subjected to compression in its own plane, it is necessary to attach to it stiffening elements which will perform one or both of two important functions:

(a) Add additional strength to the structure, particularly for the resisting of compressive loads

(b) Preserve the aerodynamic shape of the airplane when the covering sheet is loaded up to or beyond its critical buckling load

For a cylindrical structure, such as a wing or fuselage, the first criterion is satisfied by the use of stiffening elements attached to the skin and running parallel to the axis of the cylinder. These are known as axial or longitudinal stiffeners, or simply as longitudinals. The second function is performed by placing stiffening members, having the proper shape, perpendicular to the axis of the cylinder. These members tend to preserve the external shape of the structure and act as supports for the longitudinals, and they are known as ribs, bulkheads, or frames. The terms longitudinals and frames will be used in the body of this paper to denote the two classes of members discussed above. The metal covering will be spoken of as the sheet.

An inspection of this type of structure loaded, for example, by compression loads parallel to the cylinder axis, will show that there are several types of failure to which it might be subject. These types of failure are, in the order in which they will be discussed, material failure, local failure, panel failure, and general instability failure.

Material failure will occur if the sheet, longitudinals, and frames are so heavy that the structure will fail by passing the compression yield strength of the material. To determine the failing load of such a structure it merely is necessary to have a knowledge of the load distribution which, since there is no instability, will conform to the general beam theory equations, and the strength properties of the material. For airplanes, this form or cylinder will lead to such prohibitive sizes from a weight standpoint that it is a trivial case and will not be discussed further in this report.

Local failure is characterized by an instability of some small portion of either the frames or the longitudinals. Wide and thin unsupported legs on such stiffener sections may fail because of local plate buckling at comparatively low stresses. This collapse of part of a stiffener will precipitate its failure as a column and also might cause premature failure of the whole surrounding structure. The length of that portion of a stiffener, involved in local buckling, is of the same order of magnitude as the cross section dimensions, and the local buckling stress is not, in general, a function of the total length of the stiffener. Such a failure could occur in either a longitudinal or frame and might occur in any airplane regardless of size. The determination of the stress or load at which local buckling will take place involves the use of the stability equations of plates; these equations are obtainable in textbooks on elasticity. The solutions may have to be modified for plates containing ribs, such as bulb angles, but the principles involved are well known.

A panel failure is defined as one which will occur over a length of structure equal to one frame spacing and which is not caused by a local instability spreading from adjacent members. This type of failure will occur in a structure having relatively heavy frames and light longitudinals, the structure tending to act as a number of isolated, axially stiffened cylinders, each of which is one frame spacing long. Failure will occur in the curved stiffened sheet by some form of instability of the longitudinals, the magnitude of the failure load being dependent upon the column or torsional strength of the longitudinals, modified by the effect of the attached sheet. The only function of the frames in this case will be to determine the end fixity coefficient of the longitudinals. Inasmuch as theory and practice both indicate that for small diameters it is difficult to design frames which are light enough, this type of failure will occur in smaller airplanes (gross weight of 25,000 lb or less).

For the past few years manufacturers have been engaged in building airplanes of a size which leads to designs based on failures of the panel type. Although accurate theoretical treatment of the strength properties of curved stiffened panels is not yet available, it has been possible by experimental methods to design structures in which failures tended to fall in the panel instability classification. By laboratory testing of panels with a length equal to the distance between frames, and having a representative number of properly spaced longitudinals, a failing load could be obtained for the sheet stiffener combination. Ebner (reference 59), Gerard (reference 69), Hoff (reference 56), and other investigators have shown that a fairly accurate prediction of the bending strength of a stiffened cylinder is possible if the buckling strength of the panels making up the cylinder is known. One difficulty lies in the determination of the correct position of the neutral axis. However, Ebner, in his paper, gives a method of successive approximations, starting with the original moment of inertia of the cylinder, which seems to give calculated failing loads agreeing very closely with those obtained experimentally. A second difficulty arises in the determination of the end fixity of the longitudinals. An experimental method of finding this factor has been developed by Howland (as yet unpublished), which will aid in solving this problem. Utilizing the methods mentioned above, it is, therefore, possible to determine by experimental methods, the failing strength of cylinders which fall into the panel instability classification with a considerable degree of accuracy.

A fourth form of failure, which has only recently become important, is that which will be called general instability. This is a type of failure which will occur in a structure which has frames and longitudinals of such a size that both will fail simultaneously under the critical load. In other words, collapse will take place in such a manner as to destroy the load-carrying properties of all three structural elements: sheet, frames, and longitudinals. This type of failure will be found in larger airplanes in which the relative dimensions of the three structural elements are very small compared to the external dimensions of the structure.

THE STRENGTH OF UNSTIFFENED SHEETS AND CYLINDERS

A study of the failure of unstiffened, edge-supported flat sheet under compressive loads lying in the plane of the sheet, reveals that there are three critical points in the loading history. Up to some given load, called the buckling or stability limit, the sheet remains plane and then suddenly takes on a wave form, buckling perpendicular to its own plane. This buckling load is the first critical point and is calculable by the methods introduced by Bryan, and experimental results check the calculated values within very close limits.

If additional load is applied to the sheet, it will deflect at a more rapid rate than it did when plane, but will continue to resist an increased load. The theoretical treatment of this regime is difficult because the small deflection theory no longer holds and recourse must be had to a theory involving deflections of the sheet which are large compared to the sheet thickness. Calculations on this problem have been carried out by Marguerre (reference 71), Trefftz (reference 70), and Krom (reference 72). Their results give a reasonably good agreement with the small amount of experimental work available, but the range over which their theories are applicable is limited because in every case Hooke's law has been assumed to be valid and, because of combined bending and direct stress, the stress in the buckled sheet soon passes the yield point of the material. Thus the second point in the loading history is that load at which some portion of the buckled sheet enters the plastic range.

The third load of importance is the ultimate load which can be carried by the buckled sheet. Since this involves a calculation of the stress and strain relations of a sheet with large deflections and with portions of it subjected to stresses beyond the yield point, the theoretical solution of the problem is very complicated and there is available empirical or semi-empirical design information.

The above discussion of flat sheet was given because it might be logical to suppose that curved sheet loaded under axial compression might similarly have three loading regimes. However, this conclusion cannot be drawn for the general case. For complete cylinders experimental evidence indicates that there is practically no difference between the buckling and the ultimate load that can be supported. Buckling takes place very rapidly and the load-carrying ability of the specimen immediately decreases.

Another difference between the flat and the curved sheet problem is the poor agreement between theoretically and experimentally obtained buckling loads for the curved sheet. The classical buckling theories for circular cylinders under compression as given by Southwell (reference 2), Tolke and Sanden (reference 14), and Flügge (reference 15) all give values which are very considerably in excess of any experimentally obtained results.

For large radius-thickness ratios, the experimental load may not be over 20 percent of that predicted by the theoretical calculations. (See fig. 1.)

Flügge (reference 15) in the latter part of his paper endeavored to explain the discrepancy by a more careful consideration of the conditions at the end of the cylinder. He considered line support for the edges of the cylindrical sheet which allowed a change of slope but no radial displacement. This boundary condition leads to a barrel-shape or bulged cylinder, and the differential equations are of such a type as to indicate a progressive increase of the radial deformation at the center until plastic deformation sets in. However, experimental evidence indicates that the failure of such cylindrical shells under compression is not progressive but is very rapid.

Flügge, and later Donnell (reference 46), also introduced the idea of initial eccentricities to account for the differences between theory and experiment. From some unpublished work of Bollay at GALCIT on deliberately deformed cylinders, it would seem necessary to have initial eccentricities of over 10 times the sheet thickness to account for the large discrepancies between predicted and actual failing loads. This deflection could be easily detected and in some very carefully made cylinders, tested in this laboratory, no initial eccentricities which even approached this value were found, and their failing loads plotted very close to the curve of failing load against R/t shown in figure 1.

In summary, it can be said that for unstiffened cylinders under direct compression, buckling and failure are simultaneous and that loads above the buckling limit cannot be reached. Also, the theoretical treatment of the buckling problem is incorrect b a factor of as much as 5 for large radius-thickness ratios. The ratio between theoretical and experimental buckling load is a function of R/t as given in figure 1. Experiments also show that this ratio is a function of the thickness-length ratio, but, only for lengths so short that, in general, they are of no practical interest to the designer. Further tests are necessary to show the exact effect of initial eccentricities, but all evidence available at present indicates that it will probably be of second order.

The problem of the edge-supported curved panel under axial compression lies somewhere between that of the flat sheet and that of the complete cylinder. For sheets of very small curvature, it is possible to reach loads considerably in excess of the buckling load of the curved sheet. However, as the curvature increases, the difference between the buckling and the ultimate load decreases until finally a point is reached in which the curved panel behaves exactly as the complete cylinder and no additional load can be supported after buckling. This is shown by some experimental results given in a paper by Wenzek (reference 68) which are plotted in figure 2. These points indicate that, for small values of the developed width-radius ratio, the curved plate tends to act as a flat plate and carry considerable additional load beyond that causing buckling; but, for large values of this ratio, the load-carrying ability of the plate drops off immediately as soon as it has buckled. The exact laws governing this transition region are not known, and much more work must be done to clarify the parameters upon which the ultimate load of curved, edge-supported plates depend.

GENERAL INSTABILITY OF STIFFENED CYLINDERS

General Discussion

General instability is that type of cylinder collapse in which the sheet, longitudinals, and frames all fail simultaneously. Inasmuch as this type of failure involves the entire cylinder and is only to be found in relatively large airplane structures, it can readily be seen that experimental tests on structures of every particular design would be costly and difficult to make. It is, therefore, concluded that some form of theoretical solution, which has been checked by careful tests on a number of suitably designed representative specimens, is vitally necessary. This section of the report will deal with the present state of the knowledge regarding this problem.

There are two possible types of cylinder failure which fall into the class of general instability. The first of these occurs under bending loads and is characterized by a general flattening of the cylinder. This type of failure was discussed by Brazier (reference 13) for the case of the unstiffened shell under bending, and his results have been applied by some investigators to the stiffened shell problem. One of the basic assumptions involved in this solution of the problem is that the cylinder is infinitely long, and both theory and experiment indicate that, for general flattening to occur, the length-diameter ratio of the cylinder must be so large that it is completely out of the range of aircraft structures. Also, it can be shown experimentally that cylinders of a size comparable to airplane structures never fail by flattening but always fail through the development of a multilobe wave type of buckling.

The second class of general instability failure is that in which the wave form of the buckle is multilobe in nature and has, in general, a wave length less than the total length of the cylinder. This buckling form corresponds to the usual diamond-shape wave pattern which is found connected with the failure of unstiffened cylinders under compressive loads. An interesting fact is discovered upon examining the literature dealing with the general flattening and the multiwave forms of failure. Experimental observations show definitely that general flattening will never occur in cylinders unless they are extremely long compared with their diameters and that all cylinders with sizes comparable to airplane structures will fail by buckling into a number of short waves (involving one or more frames). However, the theoretical treatment of the case of general flattening gives predicted loads which are in good agreement with those found experimentally, while the theoretical treatment of the multiwave form gives predicted loads which are much higher than those obtained by experimental tests.

Two methods have been suggested for the theoretical solution of the problem of general instability. First, it might be possible to distribute the stiffnesses of the longitudinals and frames over the entire cylinder, forming an unstiffened orthotropic cylinder which then could be treated as a simple unstiffened cylindrical shell. The thickness and stiffness of this shell in the longitudinal direction would be different from that in the circumferential direction of amounts depending upon the areas and stiffnesses of the longitudinals and the frames, respectively. This rearranging of the original stiffened cylinder into an equivalent unstiffened, orthotropic cylinder will be termed the "equivalent shell" method.

A second method that could be used would be to consider the sheet, the longitudinals, and the frames as components of a statically indeterminate truss system. The longitudinals and frames, each with its proper effective width of sheet acting with it, would form the normal load-resisting members, while a suitable amount of sheet in each panel would act as a tension diagonal to transmit the shear forces. This method of analysis will be termed the "equivalent truss" method.

From elementary considerations it would seem that the first method would be more suitable in a structure in which the rigidity of the sheet covering was larger than, or at least of the same order of magnitude as, that of the longitudinals and frames. In this case, the distribution of the stiffening members to form an equivalent shell would be the same as applying a correction factor to the thickness and the rigidity of the sheet. The second method, for similar reasons, would seem to apply more accurately to structures in which the frame and the longitudinal rigidities were large compared with those of the sheet. In this case, the sheet, through the effective width acting with the stiffeners, would tend to modify the properties (such as the areas and the moments of inertia) of these members acting as elements of a truss. In addition, the sheet also would act as a tension diagonal to resist the applied shearing forces. For typical large airplane fuselage structures, it would seem that the equivalent truss method of analysis would represent more closely the actual physical picture. It is realized, however, that the truss method probably will lead to a longer and more tedious solution, and, therefore, from the standpoint of simplicity of calculation methods, it would be desirable to use the equivalent shell method if it can be made applicable.

Theoretical Treatment of the Problem

The problem of general instability is new and has had comparatively little attention from those engaged in structural research. Seven references on the subject were found. They are Taylor (reference 47), Dschou (reference 51), Timoshenko (reference 48), Nissen (reference 50), Heck (reference 53), Ryder (reference 55), and Hoff (reference 56). The papers have been studied in detail, and the theoretical results of these investigators have been checked against the meager experimental information available. All seven authors mentioned above have used the equivalent shell method of analysis, except that Hoff distributes only stiffness of longitudinals and not that of the frames. The detailed discussion of the assumptions underlying each of the seven papers will now be given.

Method of Taylor.— Taylor's paper deals with the strength of stiffened flat panels, stiffened curved panels, and the general instability of complete cylinders under bending. By use of the equivalent shell method, the rigidities of the longitudinals and the frames are distributed uniformly in the circumferential and axial directions, respectively. The differential equation of the shell is then solved, by the assumption that the deflected form is a sine wave in both directions, specifically stating that the frame spacing must be such that the wave length of the buckle will take in two or more frames. The shear in the middle plane of the shell is assumed to be taken only by the sheet, and a reduced shear modulus is used to take account of local sheet buckling.

Method of Dschou.— Dschou also assumes distributed longitudinals and frames and treats the resulting orthotropic shell. The torsional stiffness of both longitudinals and frames is assumed to be so small as to be negligible. As in the method of Taylor, the wave length of the buckled cylinder must be such as to include more than one frame but must be somewhat shorter than the total length of the cylinder. The equation obtained by Dschou for the buckling stress is set up somewhat differently than those of the other investigators in that it consists of two parts; that is, one term which corresponds to the buckling strength of the curved panel developed into a stiffened flat panel, plus a second term which is a function of the curvature. The method of solution is based on the assumption of a wave form which can be expressed by a trigonometric series. Unfortunately, the way in which the author obtains the stability criterion from the fundamental differential equation for the strain components is not discussed and cannot be followed from the published results. A number of buckling forms are discussed, and the exact equations are simplified for use in design.

Method of Timoshenko.— Timoshenko deals only with axial compression and uses the differential equations developed by Flügge (reference 15), modifying the shell properties to include the effect of the longitudinals and frames which are uniformly distributed. He specifically states that the effective width of the sheet should be used when calculating the rigidity of the stiffeners. The assumption of small deflections is made and a doubly sinusoidal wave pattern is assumed.

Method of Nissen.- This paper is mainly concerned with the correlation of some experimental tests on corrugated cylinders with the theoretical work of Dschou. The new feature involved is the determination of the longitudinal, the circumferential, and the shear rigidities by experimental methods. The longitudinal rigidity is determined by testing a section of the corrugation as an Euler column and, from the failing load, calculating an effective flexural rigidity (EI) in the longitudinal direction. By testing a section containing several frame members as a beam under bending loads, the circumferential rigidity was obtained, and, by a third experimental test, an effective shear modulus was calculated. These factors were then put into the theoretical equations of Dschou, the results being compared to those obtained experimentally. However, Nissen did not consider the whole equation of Dschou, but used only that part which corresponded to the failing stress of the developed flat panel, thus entirely neglecting the term which takes into account the effect of curvature. For this reason, his agreement on complete cylinders is poor, but his agreement with tests on curved stiffened panels is good. As the method used by Nissen is similar to the methods employed for structures falling into the panel-instability classification and involves a technique known by the present designers, it might be attractive to them. Its limits of applicability, however, are as yet unknown and the procedure therefore must be used with caution. It does not add anything toward the development of a theoretical solution of the problem which would eliminate the need for elaborate panel tests on all new designs.

Method of Heck.- Heck's paper deals only with the problem of elliptical cylinders under pure bending. The effect of the longitudinals and the frames is distributed and the sheet area is considered to be acting. The method of Brazier is used with assumes that failure will occur by a general flattening of the cylinder. It is based on the following additional assumptions:

(a) That the cylinder is infinitely long

(b) That no local (or panel) instabilities occur

(c) That all stresses remain below the proportionality limit of the material.

The small deflection theory is used in which second-order terms in the displacements are neglected; however, a discussion of the effect of neglecting these is given. Although the theoretical results give scattered agreement with experimental failing loads, the type of failure assumed definitely does not occur. In the experimental tests no general flattening could be detected, and a short wave failure always occurred. This is as would be expected, inasmuch as the experimental cylinders were only a few diameters long and did not, therefore, even remotely agree with the assumption of infinite length.

Method of Ryder.- This method is a modification and simplification of the work of Taylor and Timoshenko. In addition to the assumptions made in the above papers, Ryder multiplies the calculated loads by an additional reduction factor of 0.47/0.60 to take account of the discrepancies found between theory and experiment for the unstiffened cylinder. He also introduces a factor to account for the end fixity coefficient of the longitudinals. The work has been put into a graphical form which simplifies the use of the equations; however, the range of variables used in the charts is not always sufficient to cover all types of large airplane structures.

Method of Hoff.- Hoff's paper gives a general summary of stiffened shell theories starting with the problem of the strength of the stiffened flat panel. He also discusses at length that type of general instability treated by Brazier and Heck in which failure occurs by a general flattening of the cylinder. Using the approximate solutions of Rayleigh, Ritz, and Timoshenko, he reaches essentially the same conclusions that have been drawn by other investigators; namely, (a) that the assumption of failure by flattening gives failing moments which agree closely with those found experimentally; (b) that the calculated deflections for flattening to occur are much higher than any found experimentally; (c) that the actual deflections of the test cylinders indicate that the assumption of flattening is not valid for typical structures and will hold only for cylinders which are extremely long; and (d) that all test specimens fail by short wave buckling for which the equations developed are not applicable.

Hoff also treats the case of general instability of the second type (buckling with a wave length shorter than the length of the cylinder), using a minimum energy method of solution. His treatment differs from that of the other investigators in that he distributes the rigidities of the longitudinals but uses the frames as local elastic supports for the longitudinal elements of the shell. The contribution of the sheet to the elastic energy is neglected entirely, and it is possible that this neglect is the reason that Hoff's method is the only one which gives conservative results. However, the results are, in general, too conservative to be practically applicable, and, in addition, it is necessary in this analysis to know the number of frames which will be involved in the general instability, which feature makes it difficult to use the method in design.

Hoff introduces what he calls a structural coefficient Λ which is a function of the geometric and rigidity properties of the stiffened cylinder. By the use of this coefficient, a designer should be able to predict whether a given cylinder will tend to fail by general instability or whether it will stay in the class of panel instability failures.

Correlation of Theoretical Work and Experimental Data

In studying the available sources of information, it was found that there were practically no experimental data on cylinders which failed by general instability. Hoff reported on the testing of two cylinders under bending, one of which failed by panel instability and the other by a general instability involving the failure of two frames. The physical properties of these cylinders are given in table I, and these values have been used in the ultimate load equations of the other investigators in order to obtain, if possible, a correlation of results. Inasmuch as the equations of Timoshenko differ but little from those of Taylor, values calculated from his equations are not included in the tabulation. The methods of Heck and Nissen, based as they are on the assumption of a general flattening, which could not occur for cylinders of the length used by Hoff, are also not considered. This leaves four methods of calculating the failing load of these cylinders, and the calculated and the experimental values are given in table II.

It is seen from table II that the disagreement between theory and experiment is considerable. The disagreement for specimen 1 is to be expected, inasmuch as this specimen did not lie in the general instability regime and the large predicted values for this test have only academic interest. However, even for the specimen which failed by a general instability involving two out of five frames, the predicted values of the failing stress are all too high by factors ranging from 2 to 3, with the exception of the predicted values of Hoff, which are all too low. Actually the values which a designer would get by correctly using Hoff's method are those in column B. If, on the other hand, some means could be found to predict accurately the number of frames which would fail in a specimen, it is seen that Hoff's method might give results which would be satisfactory from a design standpoint.

Those results should not be given more consideration than they deserve. They do indicate that the methods do not check with the facts for the particular cylinders tested, but they give no indication of whether the second cylinder was in a regime in which complete general instability could be expected to occur or was in some transiton region between panel and general instability. Many more tests are necessary before it will be possible either to praise or condemn any of the methods proposed; under any circumstances, the results of table II indicate that any method must be used with considerable caution for the present.

The only other tests which were made on cylinders that were close to the general instability regime were some made on corrugated cylinders at Stanford University (reference 34). Table III gives the experimental value of the critical stresses as compared with values calculated by the methods of Taylor, Dschou, and Hoff. The method of Ryder could not be used because the properties of the cylinders were such that they were off of the range of his design charts.

With Hoff's criterion for general instability, cylinders 2 and 3 should have been in that classification. The failure, however, was of a panel instability type in all three cases. Here, again, it is found that the methods of Dschou and Taylor give predicted failing stresses which are exceedingly non-conservative, and the method of Hoff gives highly conservative values. This last series of tests shows also that Hoff's method is not always correct as to its prediction of the type of failure to be expected in a stiffened cylinder.

Guggenheim Aeronautical Laboratory,
California Institute of Technology,
November 1938.

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TABLE I

PHYSICAL DATA FOR CYLINDERS TESTED BY HOFF

Item	Specimen 1		Specimen 2	
	Considering effective width	Without considering effective width	Considering effective width	Without considering effective width
Radius, r	9.85	9.85	9.85	9.85
Skin thickness, t	.00788	----------	.00788	----------
Area stiffener, A_{st}	.0284	.0272	.0284	.0272
Moment of inertia of stiffener, I_s	.000241	.000198	.000241	.000198
Stiffener spacing, d	2.58	2.58	2.58	2.58
Area of flame, A_f	.0284	.02457	.00980	.00606
Momemt of inertia of frame, I_f	.000241	.000198	.0000325	.0000224
Frame spacing, L	7.88	7.88	7.88	7.88
$I_1 = I_s/d$	.0000934	.00007674	.0000934	.00007674
$I_2 = I_f/L$	.00003058	.0000251	.00000412	.00000284

TABLE II

COMPARISON OF DIFFERENT METHODS OF PREDICTING

THE CRITICAL STRESS OF STIFFENED CYLINDERS

UNDER PURE BENDING

Specimen	Experimental failing stress	Type of failure	Taylor	Dschou	Ryder	Hoff	
						A	B
1	29,750	Panel insta-bility	194,000	203,000	167,700	27,700	14,800
2	28,400	General insta-bility	52,300	92,400	64,500	21,300	7,300

All calculated values are based on section properties including the effective width of sheet.

(A) Using the criterion that two frames would be involved in the failure. This was actually true for specimen 2, although there would be no way of determining this fact prior to the failure of the specimen.

(B) Using the criterion that 5 frames would be involved in the instability. This would be the maximum that could be involved and, therefore, would be the value that the designer would be forced to use in lieu of better information.

TABLE III

BENDING TESTS OF CIRCULAR CYLINDERS OF

CORRUGATED ALUMINUM ALLOY SHEET

Cylinder	Experimental value of critical stress	Type of failure	Frame spacing (in.)	Calculated Euler column stress, c = 1.0	Taylor's max. stress	Dschou's max. stress	Hoff's		
							Λ	Max. stress	Max. stress
1	18,100	Panel instability	18	6,580	72,900	68,800	0.824	[1]7,300	
2	27,150	Panel instability	9	26,320	97,000	95,000	6.96	[2]11,700	[3]9,700
3	28,400	Panel instability	9	26,320	97,000	95,000	6.96	[2]11,700	[3]9,700

[1]Assuming 2 frames involved in the failure

[2]Assuming 3 frames involved in the failure

[3]Assuming 5 frames invovled in the failure

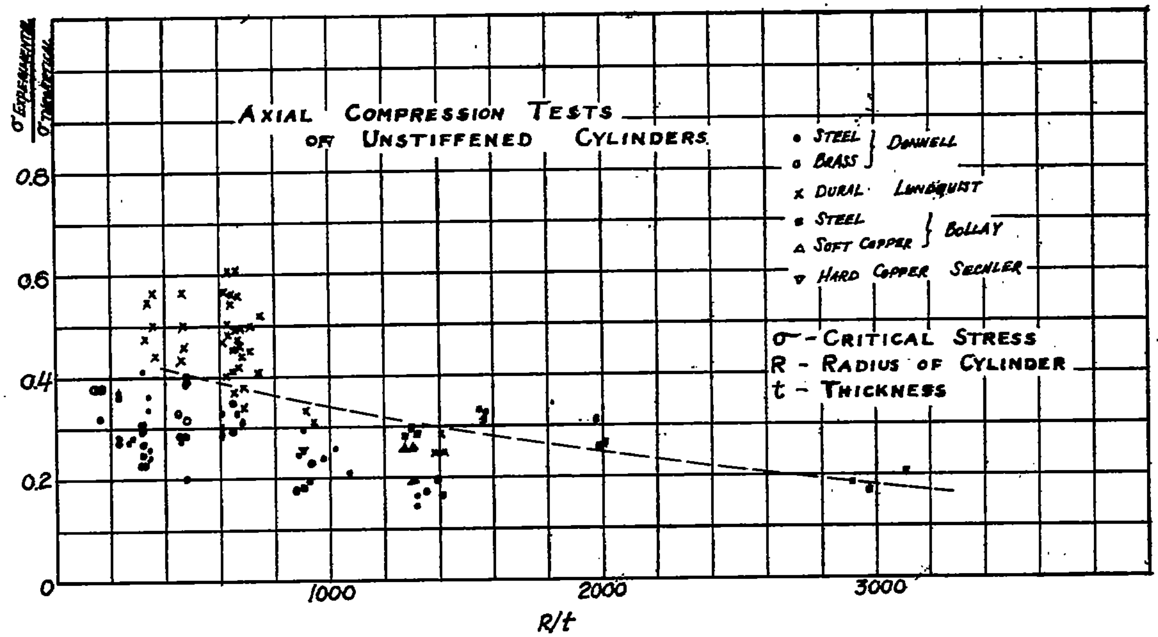

FIG. 1.

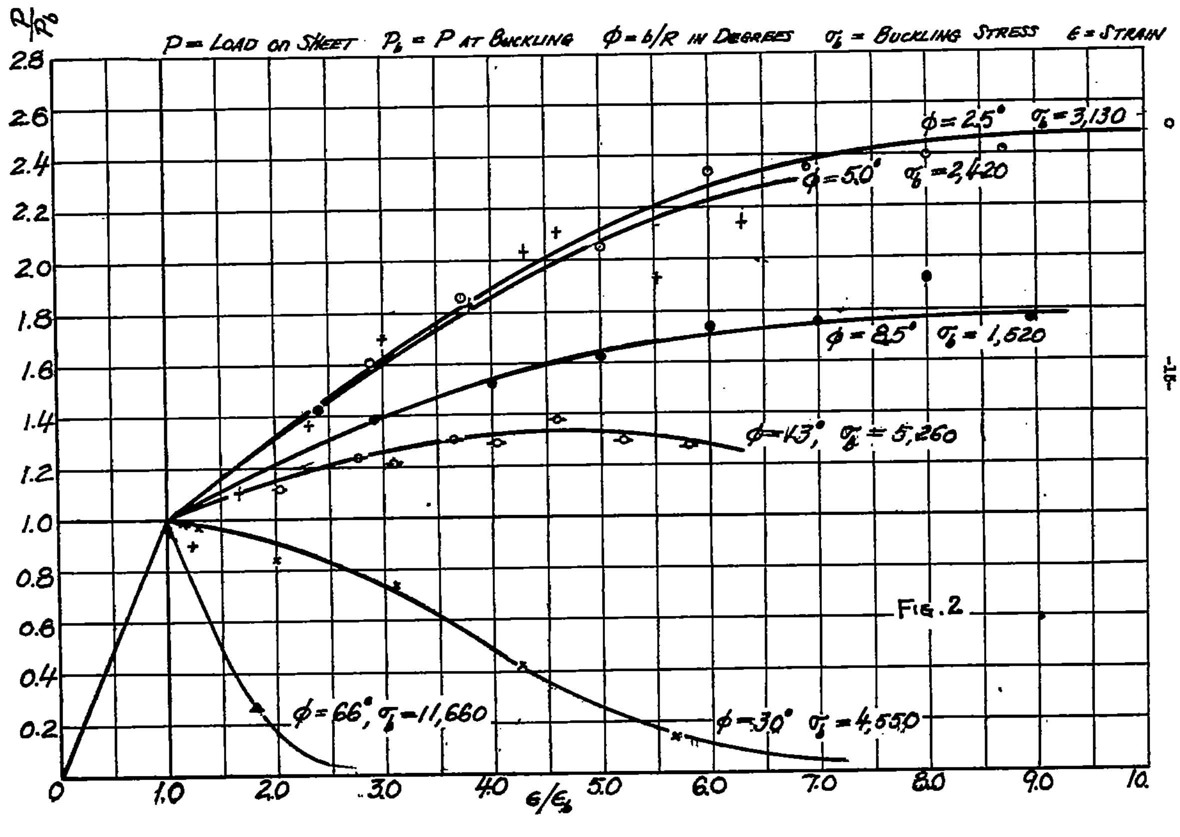

FIG. 2

-14-
-15-